Things of science

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ABACUS

ABACUS

The development of numbers and their use was a slow process in man's history. Although we use numbers freely today, the numbers concept took thousands of years to be formulated and to become a useful tool for man.

The need for some form of calculation arose as man began to understand the meaning of ownership, and it became necessary for him to count his possessions, such as sheep and cows.

At first he used sticks and pebbles to show quantities. Gradually from these beginnings, the abacus, the first calculating machine, was born.

Since the basis for counting in man's early history was his fingers, the number system was naturally based on ten (10) and is still the most widely used today.

The abacus dates back to prehistoric times. The first abacus was probably a board covered with dust on which numerals were traced. An early form of abacus was a table marked with lines on which pebbles or beads were placed to indicate quantities. Other abaci were made with grooves to hold the stones or beads.

It is known that all the early civilizations devised some sort of abacus.

As life became more complex, numerical quantities became larger. The abacus was modified, therefore, to take care of this increase in quantities, and the number of columns was increased. The columns from right to left were designated as ones, tens, hundreds, etc.

When the ones column amounted to 10, then a pebble was added to the tens column to the left and the ones column was cleared of all pebbles. When the tens column added up to 10 pebbles, a pebble was added to the hundreds column to its left and the 10 pebbles in the tens column were removed and so on. This is similar to the carrying we employ today when adding columns of numbers with pencil and paper.

The very useful zero symbol was orginated in India to indicate an empty column cleared of all pebbles. At first, a dot was used to represent an empty column. Later the zero was invented, taking the place of the dot.

The abacus is still in wide use in a number of Asiatic countries, including China and Japan.

The basic methods in arithmetic today are still the same as those used when man relied primarily on the abacus for his computations.

With the materials in this unit, you will make an abacus and learn how to add, subtract, multiply and divide on this instrument.

SOROBAN

The abacus is still in universal use in Japan, being taught in all elementary schools. The Japanese abacus is called the soroban.

Figure 1 shows the structure of the soroban. It is usually constructed of polished hard wood and bamboo, and the counters are shaped like two cones with bases attached to provide easy manipulation and speed in handling. An expert abacus operator can calculate figures at very high speed, even faster than an electrict calculating machine.

The abacus you will construct will be made from beads, wire and cardboard. But it will have the basic structure of the soroban and although you may not be able to make rapid calculations you will learn the method of calculating on an abacus. Later if you wish, you can pur-

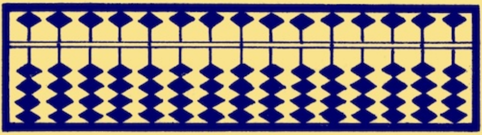


Fig. 1

chase a real soroban and with practice become proficient in its use.

First identify your specimens.

BEADS—75 colored beads.

WIRE—15 four-inch lengths of cotton-coated wire.

DIE-CUT CARDBOARD—For making the frame of the abacus.

The abacus in this unit will have 15 rows of counters, the size generally used for beginners. The standard soroban used by clerks and bankers and for other commercial purposes usually have 23 rows. The soroban in current use has one counter above the crossbar and four below it. The earlier soroban had five counters below the bar.

Experiment 1. Examine the 3- x-5-inch cardboard form in your unit.

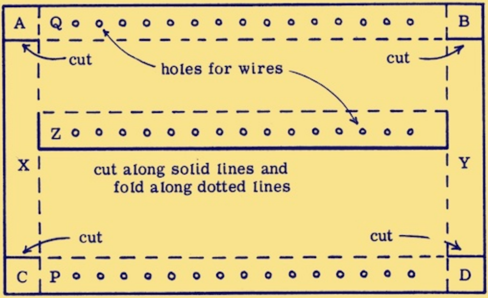


Fig. 2

Note that the dotted lines shown in Fig. 2 are scored on your cardboard piece. Fold along these lines away from the scored side.

Shape the cardboard into a box by taping or gluing corners A, B, C and D to the outside of sides X and Y.

Fold strip Z upright the length of the box and tape each end to the inside of sides X and Y, being careful not to cover up any of the holes in Z. This forms the frame for your abacus.

Next, push one end of one of your four-inch wires through the first hole in side P, beginning from either end. Insert four beads on the wire and then pass it through the first hole in Z. Add another bead and pass the wire through side Q. Fold the excess wire on each end downward against the sides and secure them firmly with tape. Be sure the part of the wire holding the beads is straight and taut (Fig. 3).

Repeat the procedure until all 15 wires with their beads are placed in position. The wires represent the rods of the soroban and the beads, the counters.

Familiarize yourself first with the parts of the soroban (Fig. 4).

The counters (beads) above the crossbar or bar represent 5, 50, 500, 5,000,

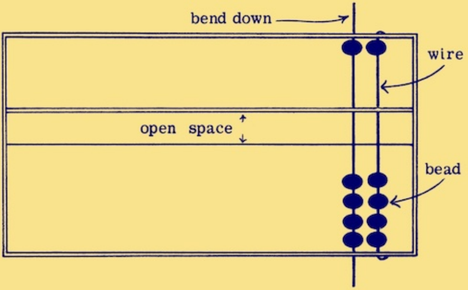


Fig. 3

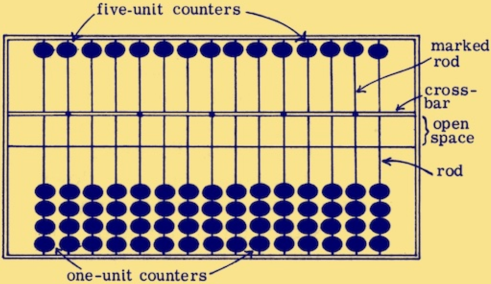


Fig. 4

etc. and are called five-unit counters. The counters below the bar represent 1, 10, 100, 1,000, etc. and are termed one-unit counters.

With red ink or pencil mark the bar with a dot at the 5 positions shown in Fig. 4. The rods at these positions are

designated as "marked rods." These dots serve as guides for the placement of numbers. They also may be used to indicate the position of decimal points, or the comma in figures such as thousands and millions.

To operate the abacus you must first have all the counters in their zero positions, or away from the bar as in Fig. 4. After finishing a calculation and before starting a new one, you should always clear the abacus to show zero on all rods. Work with the abacus flat on the desk.

In moving counters on a single rod, the five-unit counters are pushed upward or away from you with the tip of the index finger and the one-unit counters, by means of the thumb. To move both one-unit and five-unit counters downward or toward you, the index finger is used. If this is difficult to do with your small abacus, use the tip of a pencil to push the beads up and down.

As you all know, in a whole number, the digit to the extreme right represents the "ones," the next digit to the left, the "tens" and the next digit to its left, the "hundreds" and so on. For example, 2,362 is

thousands	hundreds	tens	ones
2	3	6	2

These designations are used in referring to the numbers on the abacus also.

Experiment 2. To indicate numbers, the counters are moved against the bar. You can enter numbers in any series of rods. However, for convenience, a marked rod is usually used as the "ones" position of a number.

To indicate 1, push 1 one-unit counter on a marked rod up to the bar.

To make 2, push up another counter on the same rod. Add 2 more one-unit counters to make 4.

To add 5 to the 4, push down the five-unit counter on the same rod. You now have a total of 9 (Fig. 5a). A total of 9 is as much as you can show on a single rod.

Form 6, 7 and 8 in sequence. Each of the numbers 6 through 9 can be formed in a single operation by pushing down the five-unit counter and moving up the one-unit counters at the same time.

Remember to clear the whole abacus

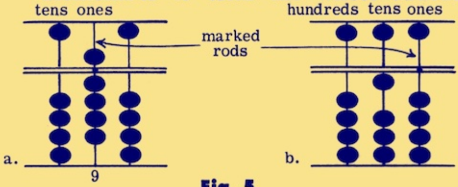


Fig. 5

to zero each time you start a new calculation.

Experiment 3. To show 10, raise 1 one-unit counter to the bar on the first rod to the left of the marked rod or on the "tens" rod (Fig. 5b).

To show 100, clear the tens rod and push up 1 one-unit counter on the next rod to the left, the "hundreds" column.

Form 2,653 (Fig. 6).

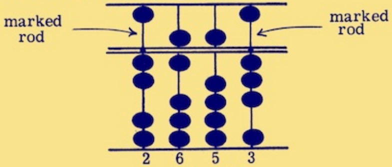


Fig. 6

The mark on the thousands rod indicates the position of the comma.

Now that you know the system, form the following numbers: 1,008; 395; 9,603; 422; 751; 1,390.

ADDITION

Experiment 4. Add $2 + 1 = 3$.

Select any marked rod and push 2 one-unit counters up to the bar. To add 1, enter 1 one-unit counter on the same rod with the thumb. (Fig. 7).

Problems: Add $2 + 2$; $3 + 1$.

Experiment 5. Add $5 + 2 = 7$.

First move down the five-unit counter

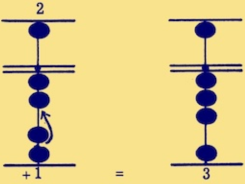


Fig. 7

into position against the bar with your index finger. To add 2, move 2 one-unit counters up to the bar on the same rod with your thumb.

Experiment 6. Whenever we add with paper and pencil, we start with the column of digits at the extreme right. In addition with the soroban, we start with the digits at the extreme left.

Add two-digit numbers in which no carrying is required. That is, the sum of the numbers in a single column is less than ten.

Add $23 + 15 = 38$.

When numbers of more than one digit are added, each column of digits is considered separately. The addition is completed in the hundreds column for example, then in the tens column and then in the ones.

Place counters to show 23 (Fig. 8a) with the 3 on a marked rod. To add 15, first add the 10 of the 15. To do this,

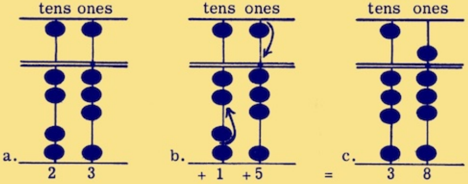


Fig. 8

move up 1 one-unit counter in the tens position (Fig. 8b).

Next add the 5 of the 15 by bringing down the five-unit counter on the ones rod (Fig. 8b). You have the sum 38 (Fig. 8c).

Add these numbers for practice: 21 + 25; 16 + 72; 74 + 25; 39 + 50.

Experiment 7. Sometimes there are not enough unused one-unit counters left on a rod to take care of the number to be added. Then the procedure is slightly different. For example, 2 + 4 = 6.

Place the 2 in position (Fig. 9a). Since you have used up 2 one-unit count-

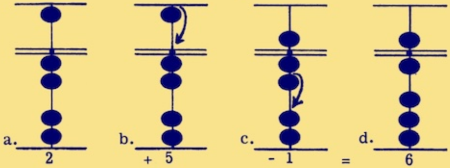


Fig. 9

ers, you will not be able to add 4 one-unit counters to the 2 (Fig. 9a). Therefore, first bring down the five-unit counter on the same rod (Fig. 9b). But, in doing so you have a total of 7, since you have added 1 too many. To take care of the excess, remove 1 one-unit counter on the same rod (Fig. 9c). Now the abacus should show a total of 6 (Fig. 9d).

Always add the 5 first and then remove the one-unit counters in such calculations taking place on the same rod. If you run your finger down the rod, you will understand the reason for this sequence. With one continuous stroke, you can push down the five-unit counter and then any excess one-unit counters below.

Experiment 8. Add two-digit numbers that meet similar conditions as in Experiment 7.

Add 33 + 34.

First place 33 in position.

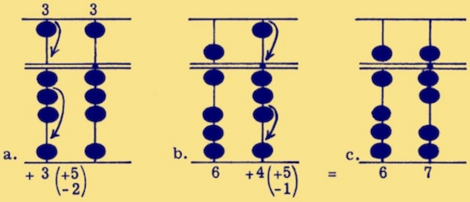


Fig. 10

To add 34, first add the tens, or 30. As you can see, you can't add the 3 to the tens column by using the one-unit counters, so add 5 which is 2 more than needed. Next, remove 2 one-unit counters (Fig. 10a).

Now add the ones, the 4 of 34 and the 3 of 33. You can't add 4 one-unit counters so what must you do (Fig. 10b)?

The sum should be 67 (Fig. 10c).

Practice problems: 13 + 44; 21 + 34; 43 + 22; 24 +34.

Experiment 9. Add 3 + 7.

Enter 3 one-unit counters on a marked rod (Fig. 11a).

Add 7. You know you can't add 7 to a rod in which 3 one-unit counters have already been used since the sum 7 + 3 is greater than 9. This means you will have to carry to the next column to the left, or the tens column.

To do this, ask yourself how much you must add to the 7 in order to make 10. You know the answer is 3, so take away 3 one-unit counters from the ones column, leaving zero (Fig. 11b). Mentally add the 3 that you have subtracted from the ones column to the 7 to be added in your problem, which equals 10. Enter this 10 into the tens column by moving up 1 one-unit counter in the tens rod

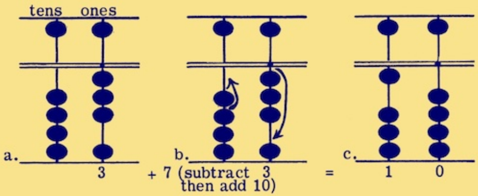


Fig. 11

(Fig. 11b). The abacus now shows the sum of $3 + 7 = 10$.

The sequence is to remove the 3 one-unit counters first and then add the 1 to the tens column. This sequence also adds to speed. Note that the position of your fingers allows you to accomplish the addition and subtraction in a single operation. Push down the 3 one-unit counters with the index finger and at the same time move up one 1-unit counter in the next column with your thumb.

The arithmetical process is exactly the same as when the addition is done with paper and pencil.

To add $\begin{array}{r} 3 \\ +\ 7 \\ \hline 10 \end{array}$ you remove the 3 and 7 (or the sum of 10) from the ones column, leaving zero and add 1 to the tens column.

Experiment 10. Add $7 + 5$.

Enter 7 on a marked rod. Now add 5.

You know that you can't add 5 on a rod which already has 7, so think what you must add to 5 to make the ten you must carry to the tens column. Continue as in Experiment 9. Do you get 12?

Add $9 + 7$; $6 + 4$.

Experiment 11. In certain cases an extra step is involved.

Add $6 + 7$.

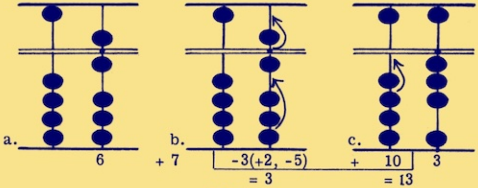


Fig. 12

Place 6 in position (Fig. 12a).

You can't add 7 to the 6 on the same rod so think $7 + ? = 10$; $= 3$. Subtract this 3 from the 6 (Fig. 12b). However, since you can't remove 3 one-unit counters because only one is used in making 6, mentally subtract the 3 from the 5 of the five-unit counter used in making the 6 instead. Enter the difference of 2 in the ones column and then remove the 5 (Fig. 12b). Always add the difference first and then subtract the 5. Why? Does this procedure contribute to speed?

By adding 2 and removing 5 you have actually subtracted 3. This 3 plus the 7 to be added in your problem makes 10. So push up 1 one-unit counter in the tens column. The answer should be 13 (Fig. 12c). What you did was to subtract 3 and add 10 to make the 7 you must add.

Add 6 + 8; 8 + 6; 5+ 7.

Experiment 12. Now let us add larger numbers. Add 24 + 87.

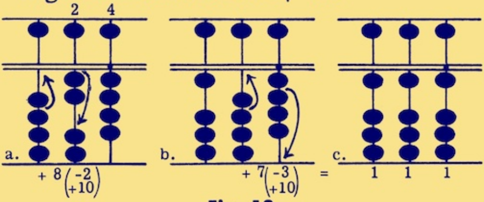


Fig. 13

First place 24 in position.

Now add the tens first.

Can you add the 8 of 87 to the 2 of the 24? If not, think 8 + ? = 10; = 2. Remove the 2 from the tens rod and add it mentally to the 8 to make 10. Push up 1 one-unit counter on the next rod to the left, the hundreds rod (Fig. 13a).

Next add the ones. Since you can't add the 7 of 87 to the ones column which already has 4, what do you do? Think 7 + ? = 10; = 3. Subtract the 3 from the ones column and proceed as before (Fig.

13b). The answer is 111 (Fig. 13c).

Add: 14 + 91; 32 + 88; 43 + 79; 32 + 98; 89 + 56; 56 + 59; 96 + 89; 83 + 78; 69 + 97.

Experiment 13. Add 67 + 66; 76 + 76; 46 + 7.

To do these additions refer back to Experiment 11.

Experiment 14. Add 798 + 6.

To add 6 to the 8 of 798, think 6 + 4 = 10. Subtract 4 from the ones rod by adding 1 first and then removing 5. Then add 1 to the tens column.

Note, however, that you can't add 1 to the 9 in the tens column. Since 1 + 9 = 10, remove 9 from the tens column and add 1 to the hundreds column. Did you get the correct answer?

Add 193 + 7; 996 + 5; 397 + 8.

Experiment 15. To add columns, place the top edge of your abacus right beneath the numbers successively, adding each succeeding number to the previous total.

Add:	66	39	569	8643
	76	22	334	367
	54	81	931	2919
	89	45	876	198
	95	93	439	5064
	32	66	215	873
	—	—	——	——

Add the first column. This is how to proceed:

Place the 66 in position. Add 76 to the 66. Move the abacus down to show 54 and add it to the previous total of 142; you get 196. Move the abacus down to show 89 and add this to the 196, and so on down to the bottom of the column.

Add the other columns.

Experiment 16. Add decimal numbers. 65.19 + 34.83

To add decimal figures, use the same procedure as in regular addition using a marked rod for the ones column. The dot indicates the position of the decimal point which is at the right of the ones column (Fig. 14).

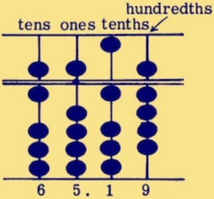


Fig. 14

SUBTRACTION

To subtract you reverse the addition process.

Experiment 17. Subtract 3 — 1.

Place 3 in position. To subtract 1, push down 1 one-unit counter, leaving 2.

Experiment 18. Subtract two-digit numbers, for example, 46 — 11.

To subtract, first remove the 10 of 11 (Fig. 15). Then push down the 1 of 11. You subtract from left to right. Is the difference 35?

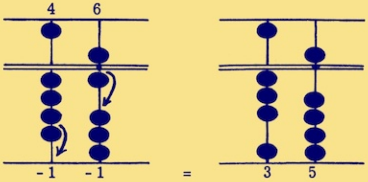


Fig. 15

Subtract 78 — 23; 84 — 32; 46 — 31.

Experiment 19. Subtract 12 — 4.

Form 12. You can't subtract 4 from the

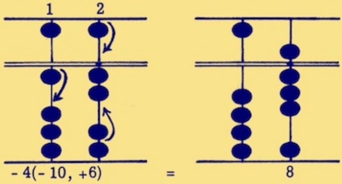


Fig. 16

2 of 12. so think 10 — 4 is 6. Remove 10 and add 6 to the ones rod (Fig. 16). You have the answer 8. Always remove the 10

first and then add the difference in this type of operation ($-10+6=-4$).

Experiment 20. Subtract 8 — 4. This subtraction requires an extra step.

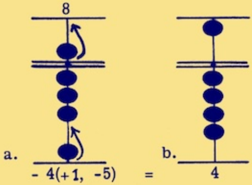


Fig. 17

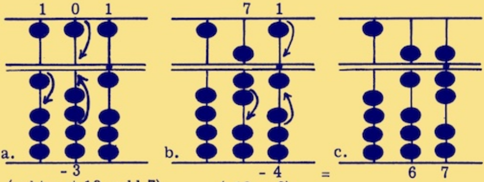
In subtracting 4 one-unit counters from 8, note that there are only 3 one-unit counters available. Therefore, mentally subtract 4 from the 5. Add the difference of 1 to the ones column first, then remove the 5. In doing this you have actually subtracted 4 (Fig. 17a). The answer should be 4 (Fig. 17b).

Experiment 21. Subtract 101 — 34. Enter 101 in position.

First subtract the 30 of 34. Since you can't subtract 3 from the zero in the tens column, go to the 1 on the hundreds rod and think of it as 10. Remove the 1 from the hundreds rod and mentally subtracting 3 from this ten, add the difference of 7 to the tens rod. In this procedure, remove the 1 first from the hundreds rod

and then add the 7 to the tens rod (Fig. 18a).

Proceed with the subtraction of the 4 of 34 in the same way. Since you can't subtract 4 from 1, think of 10 — 4. Remove 1 one-unit counter from the tens column and then add the difference of 6 to the ones rod (Fig. 18b).



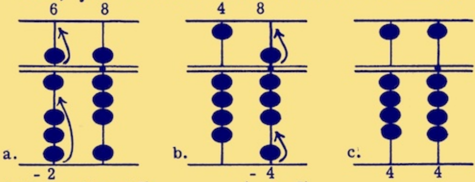
(subtract 10, add 7) (-10, +6)

Fig. 18

Experiment 22. Subtract 68 — 24.

Enter 68.

First consider the tens. You can't subtract the 2 of the 24 from the 6 of 68 by using the one-unit counters. Why? Therefore, you must think 5 minus 2 = 3.



(add 3, subtract 5) (+1, -5)

Fig. 19

Push up 3 one-unit counters (Fig. 19a) and remove the 5 (or 3 — 5 = — 2).

Now calculate the ones. To subtract the 4 of 24 from the 8 of 68, proceed as in Experiment 20 (Fig. 19b). What is your answer?

Experiment 23. Do the following subtraction problems.

(a) 122 — 89; 103 — 89; 114 — 90; 112 — 79; 172 — 98; 185 — 96.

(b) 261 — 36 — 42 — 96; 163 — 75 — 29 — 32; 321 — 92 — 63 — 32.

(c) 56 — 21 + 64 — 24; 77 — 42 + 84 — 25; 85 — 54 + 24 — 21.

MULTIPLICATION

In order to multiply and divide with the soroban you must know the multiplication table, 1 through 9, from memory.

There are two methods of multiplication with the abacus: "multiplication starting from the right" and "multiplication starting from the left."

In multiplication from the right, the procedure is to start multiplication beginning with the digit at the extreme right of both multiplier and multiplicand and continue to the left. This is the same as the usual procedure used in multiplying with pencil and paper.

This method is used mainly with the

abacus when the multiplier is a one-digit number.

In the illustrations following, counters are omitted from rods not involved in the calculation.

Experiment 24. Multiply 3 (multiplier) x 6 (multiplicand).

Place the multiplier (first factor) 3 on the second marked rod from the left of the abacus. Enter the multiplicand (second factor) 6 on the first marked rod to the right of the multiplier.

From your multiplication table you know that 3 x 6 is 18. Enter the product in the two rods to the right of the multiplicand. Then clear the multiplicand and the product of 18 is left. The ones column is the second rod to the right of the multiplicand (Fig. 20).

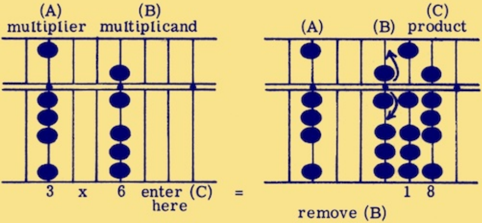


Fig. 20

Experiment 25. Multiply 7 x 18. Place the multiplier 7 and multiplicand

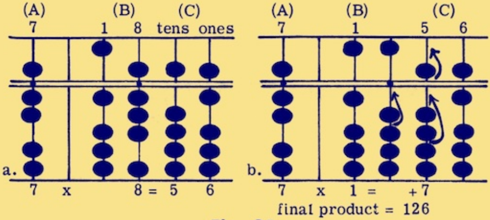


Fig. 21

18 in their positions on the abacus.

First multiply the 7 times the 8 of 18; = 56. Enter 56 on the two rods next to the 8 of the multiplicand (Fig. 21a).

Remove the 8 of the multiplicand.

Next, multiply 7 × the 1 of 18. Add the product 7 to the rod second to the right of the multiplicand 1, since this rod has become the ones position for the multiplicand being used (Fig. 21b). In the final product, you will see that this position is actually the tens column. Explain.

Remove the 1 of the multiplicand. Is the product 126?

Multiply the following: 3 × 47; 4 × 24; 6 × 37; 8 × 92; 5 × 69.

Experiment 26. Multiply 6 × 893.

Proceed in the multiplication as in Experiment 25, except that you now have three digits in the multiplicand. Remember that the ones position for each new product is the second rod to the right of

the multiplicand in use.

Multiply 4 × 407; 5 × 663; 7 × 260.

Experiment 27. Multiply 48 × 36.

Multiplication can be done in any order (Fig. 22).

When the multiplier contains two or more digits, the method, "multiplication from the left" (Fig. 22c) is usually used. This procedure is more convenient for larger numbers. As you multiply 48 × 36 on the abacus the reason for applying this

(a)

```
  36
 x48
 ---
  48 --  8 x 6
 240 --  8 x 30
 240 -- 40 x 6
1200 -- 40 x 30
----
1728
```

(b)

```
  36
 x48
 ---
1200 -- 40 x 30
 240 -- 40 x 6
 240 --  8 x 30
  48 --  8 x 6
----
1728
```

(c)

```
  36
 x48
 ---
 240 -- 40 x 6
  48 --  8 x 6
1200 -- 40 x 30
 240 --  8 x 30
----
1728
```

Fig. 22

method will become clear to you.

Place 48, the multiplier, and 36, the multiplicand, in their proper positions on the abacus (Fig. 23a).

First, multiply the 4 of 48 times the 6 of 36; = 24. Enter the 24 on the two rods to the right of the 6 of the multiplicand, the tens and ones positions for this product (Fig. 23a).

Next multiply the 8 of 48 times the 6 of 36; = 48. Add the product 48 to the second and third rods to the right of the multiplicand, the tens and ones positions

of this product (Fig. 23b).

Remove the 6 of the multiplicand (Fig. 23b).

Now multiply the 4 of 48 times the 3 of 36; = 12. Add the 12 to the two rods next to the multiplicand 3, the tens and ones positions for this product (Fig 23c).

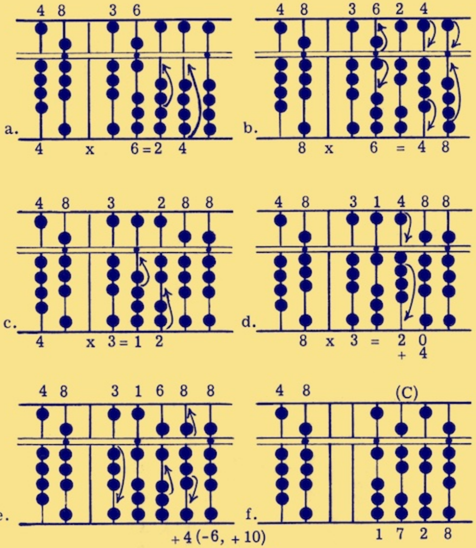


Fig. 23

Finally, multiply the 8 of 48 times the 3; = 24. Add the 24 (in two steps) to

the second and third rods to the right of the multiplicand 3 (Fig. 23d and e), the tens and ones positions for this product.

Clear the multiplicand 3 (Fig. 23e).

The product is 1728 (Fig. 23f).

Observe that in this method you clear the multiplicand step by step from right to left, thus making room for the product as it increases.

Study Fig. 23 and you can see that the thousands, hundreds, tens and ones in the final product fall into place automatically by placing each product in its proper position and adding it to the previous total as you go along. Do you see why this happens? Analyze this, writing out each product in full if necessary.

Multiply 65 × 43; 61 × 34; 21 × 54; 48 × 26; 79 × 67; 36 × 89.

The important thing is to remember the correct position for each successive product in relation to the multiplicand.

Experiment 28. Multiply three-digit

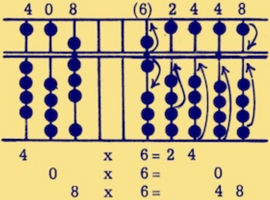


Fig. 24

numbers.

Multiply 408 (multiplier) × 6 (multiplicand).

When adding the 48, be sure to allow for the 0 (Fig. 24).

Clear the multiplicand. Answer = 2, 448.

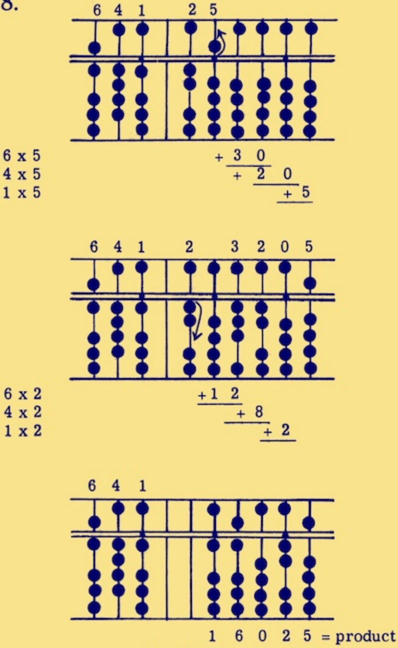


Fig. 25

Experiment 29. Multiply 641 × 25 = 16,025 (Fig. 25).

Add each new product in succession in the correct columns.

Clear each number of the multiplicand in sequence.

Multiply 635× 98; 719 ×37; 816 × 46; 527 × 523; 448 × 814.

If the multiplicand consists of 3 or more digits leave more space between it and the multiplier.

DIVISION

Division is accomplished by multiplication and subtraction.

Experiment 30. Do this problem: 9 ÷ 3.

9 is the dividend and 3 the divisor.

On the abacus the divisor is placed to the left, the dividend to the right and the quotient in between. Therefore, allow sufficient space between the divisor and the dividend (see Fig. 26).

There are certain basic rules that must be followed. If the first digit of the

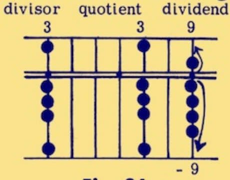


Fig. 26

dividend is larger than the first digit of the divisor, then the first digit of the quotient is placed on the second rod to the left of the dividend.

If the first digit of the divisor is greater than the first digit of the dividend, then the first digit of the quotient is placed on the first rod to the left of the dividend.

The illustration below will demonstrate the reason for the above positions of the first digit of the quotient.

$$\text{(a)} \quad 3\,\overline{)\,93}^{\;31} \qquad\qquad \text{(b)} \quad 3\,\overline{)\,126}^{\;42}$$

In (a) the first digit of the dividend is greater than the first digit of the divisor. In (b) the first digit of the dividend is smaller than that of the divisor.

To find the quotient of 9 divided by 3 ask yourself $3 \times ? = 9$; $=3$. Enter 3 as the quotient in the rod second from the left of the dividend. Why?

Multiply the quotient times the divisor and subtract the product from the dividend. $3 \times 3 = 9$; 9 from $9 = 0$.

Experiment 31. Do this division: $30 \div 5$.

Where will you place the quotient in this problem?

To find the quotient, think $5 \times ? = 30$. Place the answer 6 in position as the quotient (Fig. 27).

Multiply 6 × 5. Subtract the product 30 from the dividend leaving zero.

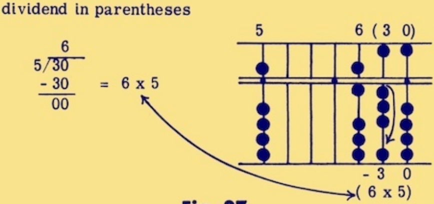


Fig. 27

Experiment 32. Do this problem: 69 ÷ 3.

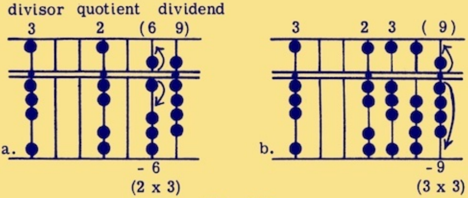


Fig. 28

Note that the first digit of the dividend is larger than that of the divisor. Where will you place the first digit of the quotient?

To divide think 3 × ? =6; = 2. Push up 2 one-unit counters in the second rod to the left of the dividend (Fig. 28a). Multiply 2 (quotient) × 3 (divisor) and subtract the product 6 from the first digit of the dividend (Fig. 28a).

Now consider the remaining digit of the dividend 9. Note that 9 is greater than the divisor 3. To find the quotient of $9 \div 3$, think $3 \times ? = 9; = 3$. Enter the 3 on the second rod to the left of the dividend 9 (Fig. 28b). Multiply 3 (quotient) $\times$ 3 (divisor). Subtract the product 9 from the dividend 9, leaving zero. Note that for this subtraction, the ones position is always the second rod to the right of the quotient.

Note that as the division proceeds, the dividend is eliminated step by step.

Experiment 33. Demonstrate that the basic procedure for the division you have just performed on the abacus is the same as the procedure followed in division with paper and pencil.

Experiment 34. Do this problem: $162 \div 6$.

Place the divisor and dividend in their positions.

Take the first two digits of the divi-

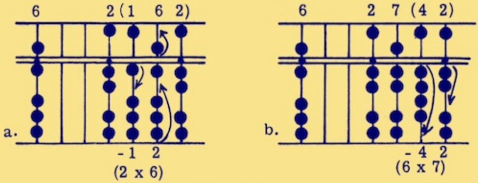


Fig. 29

dend 16. Think of the nearest multiple of 6 that will be less than or equal to 16. The answer is 2, since $2 \times 6 = 12$. Enter the 2 in the quotient position (Fig. 29a).

Multiply the 2 times the divisor 6. Subtract the product 12 from the first two digits of the dividend, the tens and ones positions (Fig. 29a). This leaves 42 in the dividend.

Now find the quotient of $42 \div 6$. Think $6 \times ? = 42$; $= 7$. Since the first digit of this dividend is smaller than the divisor, place the quotient 7 in the first rod to the left of the dividend. Multiply 7×6. Subtract the product, 42, from the dividend leaving zero.

Do these problems: $424 \div 4$; $655 \div 5$; $3{,}328 \div 4$; $4{,}914 \div 7$; $1{,}539 \div 9$.

Experiment 35. Do this problem: $65 \div 4$.

The quotient is 16 and a remainder of 1. Can you demonstrate this calculation by diagram?

Experiment 36. Obtain a decimal number as a quotient. Try $135 \div 6$. Can you arrive at the quotient 22.5?

Experiment 37. Divide by two-digit numbers. For example, $4{,}428 \div 54$.

Place the divisor and dividend in their positions.

If you wish, do the division as given below on paper first to visualize what takes place in the procedure. Then do the calculation by abacus.

When the divisor is a two-digit number, mark off the dividend beginning from the left into groups of two digits. The first group is 44. Ask yourself, what times 5 will equal 44 or less; = 8.

Place the 8 in the quotient in the first rod to the left of the dividend. Multiply 8 times the 5 of the divisor 54 and subtract the 40 from the first two digits of the dividend.

Next, multiply the quotient 8 times the 4 of the divisor; = 32. Subtract the 32 from the first two digits of the remaining dividend 428. You now have 108 left in the dividend.

Divide the first two digits of the remaining dividend, or 10, by the 5 of the divisor; = 2. Enter the quotient 2 in the first rod to the left of the dividend 108. Multiply 2 times the 5 of the 54 and subtract the product 10 from the first two digits of the dividend, 108, leaving 8. Multiply 2 times the 4 of 54 and subtract the product 8 from the remaining dividend; = 0. The quotient is 82.

Do these problems: $2816 \div 44$; $2835 \div 45$; $2016 \div 32$; $3638 \div 17$.

Division with the soroban is more complicated than the other arithmetical calculations. Here we have included the simpler problems only. However, the basic principle is the same as in the more involved divisions.

Experiment 38. The traditional abacus is operated on the number system based on ten. However, an abacus may be used with any grouping of numbers as its base.

Can you design a system for the abacus using base 8?

For those of you who wish to study the soroban further, the following references will be helpful:

The Japanese Abacus Explained, Y. Yoshino, Dover Publications, N. Y.

Soroban, Japan Chamber of Commerce and Industry, Charles E. Tuttle Co., Rutland, Vt.

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